STAGNATION POINTS: ANALYTICAL – THEORIC APPROACH

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ABSTRACT

Stagnation points in the aquifer flow field are often a sort of separation region between different velocity areas. They are generally located in the middle of an area where water circulation is limited and flow velocity is quite zero.
The presence of a stagnation point usually indicates a special condition of stability about water supplies, where the limited supply of oxygenated water and negative redox potential also can create some bad conditions, but relatively protected from other forms of pollution.
In this note some rules for a fast location of stagnation points due to withdrawal of two extraction wells are developed. The method can avoid a numerical or analytical formulation. Both analytical method and theoretical one linked to a vector velocity field interacting to groundwater during pumping well barrier are shown.

1 INTRODUCTION

Groundwater stagnation point location plays an important role as shown by many Authors (i.e. Kasenow, 1968; Winter, 1976; M.P.Anderson e al., 1992); they represents a zero flux and equilibrium points where applied forces are balanced in all direction. The behavior of stagnation points seems fundamental to understand how delineate different flow regions designed by capture curve of extraction wells.
For example, as shown in Figure 1, the zone between two rivers in the valley or in the prealpine area is particularly exposed to the establishment of areas of stagnation. The stagnation can occur, because a large area from which the flows diverge and then up towards the top is formed.

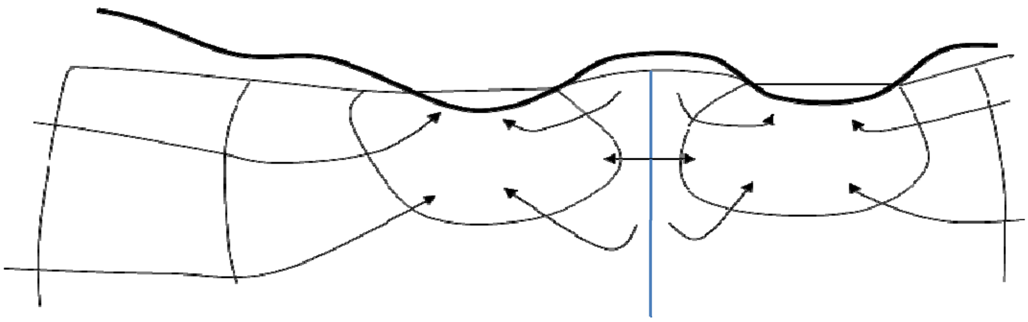

Fig. 1 – Formation of a stagnation point between two alluvial areas

The analysis and the study about stagnation points and their possible practical applications have been continued for many years by many researchers (Muskat, 1946; Bear, 1979; Javandel & Tsang, 1986; Strack, 1989; Zhan, 1999; Christ & Goltz, 2002; Christ, 2004).
Winter (1976), for example, has shown that the rate of charge of a lake occurs in the presence of stagnation points. If the stagnation point exists (Fig. 2), it is located below the bank of the lake and it is the more frequently the more the aquifers are shallow with a ratio between horizontal and vertical permeability less than 100. Since many lakes are fed by springs the formation of stagnation points does not represent an anomaly.

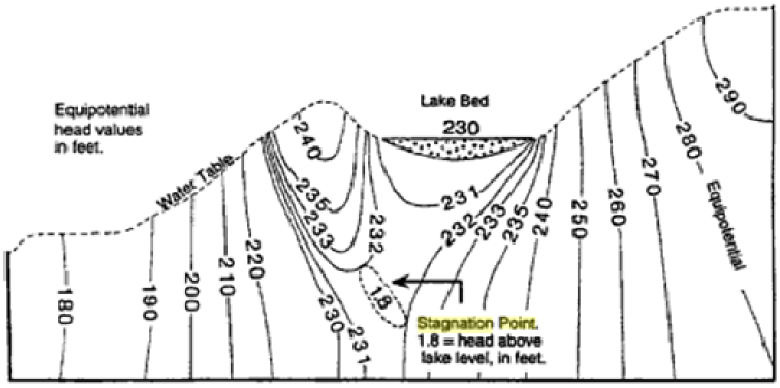

Fig. 2 – Presence of a stagnation points under the bank of a lake (from: Applied Ground Water Hydrology and well Hydraulics by M. Kasenow)

The knowledge of the location of these points at zero velocity is very important for example for the positioning of drainage systems (Fig. 3). It is observed indeed that if the gravity drain is positioned in an area at zero velocity it is not able to reduce the inflow of water towards the surface. By these characteristics, the infiltration of water in depth is evidently obstructed.

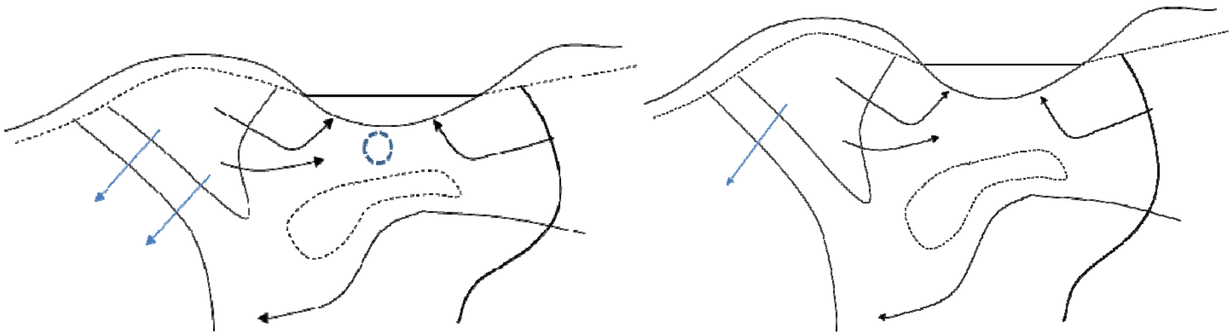

Fig. 3 – Difficulties in projecting a drainage system due to the presence of a zero – velocity domain nearby the same drain

It is possible to find out and outline these zones actuating the flow network reconstruction, helping with a piezometric map that underline the piezometric level distribution on the vertical till interested depth. It is important to understand what are the physical phenomena whose control origin and presence of zero-velocity points.

2 THEORETICAL APPROACH TO THE STUDY OF ORIGIN OF STAGNATION POINTS ALONG THE DEPRESSION DUE TO THE ACTION OF TWO WELLS

Stagnation points grow up as the equilibrium point between the velocity vector of the groundwater kJ and the vector of piezometric depression kj. If J is equal and opposite to j (i.e. if it is a vector which has the same direction but opposite orientation), their point of application is a stagnation point. The location of a stagnation point by fictitious radius of influence R0 computing is well known.

If there is a system of two simultaneous pumping wells, with the same rate of pumping, acting in an homogeneous artesian aquifer (see Fig. 4), it is possible to evaluate the position of the stagnation point in function of the parameters of the aquifer and the flows of the wells. The influence radius has finite dimension in nature, due the fact that the infiltration of rain and surface waters usually not allow an expansion of piezometric depression over few hundreds of meters.

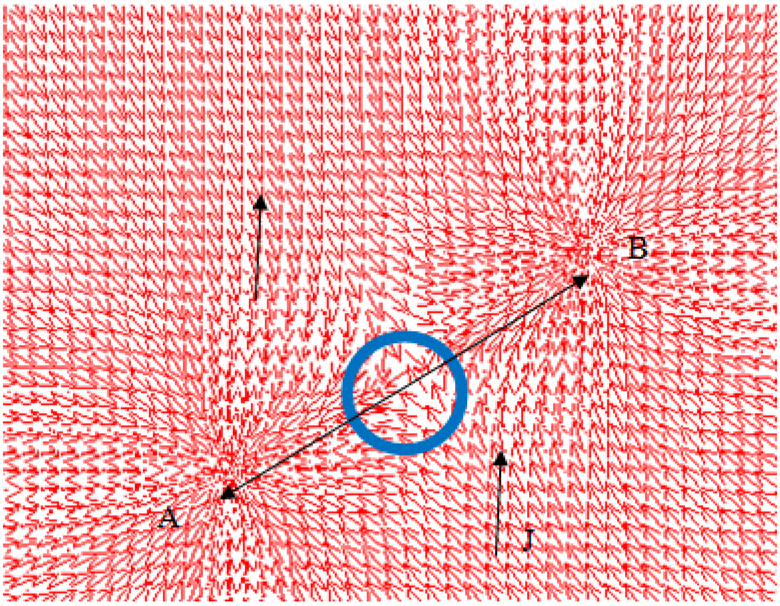

Fig. 4 – Action of two wells with the same pumping rate (Q = 3 l/s). It is possible to see a stagnation point, where the fluid has velocity equal to zero (in the blue circle, vectors have the same direction and opposite orientation)

Assuming to be in these conditions, and the two extraction wells have the same flow rate: the stagnation point is located at the intersection between the imaginary line which joins wells and their perpendicular center line.

The piezometric depressions formed by two pumping wells occur when the wells overlap a portion of the effective depression, and the influence radius of each well must to be equal at least to the half distance between wells. This distance is minimal along the line which joins the wells: R (influence radius of each well) must be equal to E (half distance between wells).

The amplitude E of the capture front of one well, must be equal to half of the capture front F, theorically located at the distance of the influence radius. In this case, according to known relations between geometric elements of piezometric depression created by inclined aquifer well:

$$E = Q/2q \; ; \; q = T_j \; ; \; R_0 = 2E/\pi \qquad \textbf{(1.1)}$$

For example, in Fig. 5, it is possible to see that only when the flow rate of the wells creates an influence radius bigger than the diameter Eeff, equal to the half distance between wells, it is possible to have a piezometric depression which sums the effects of two consecutive wells.

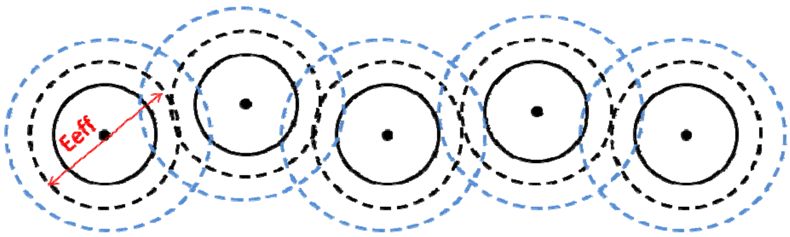

Fig. 5 – Series of wells not in axis; the piezometric depression is equal to the sum of component of adjacent wells

Examinating the simple case of two wells, a stagnation point grows up only when the sum of groundwater velocity vector , and of the vectors of the groundwater velocity due to pumping wells, generates a vector, equal and inverse of the one of the groundwater . The velocity vector towards the well is done by the sum of two components: j' and j'', one for each well.

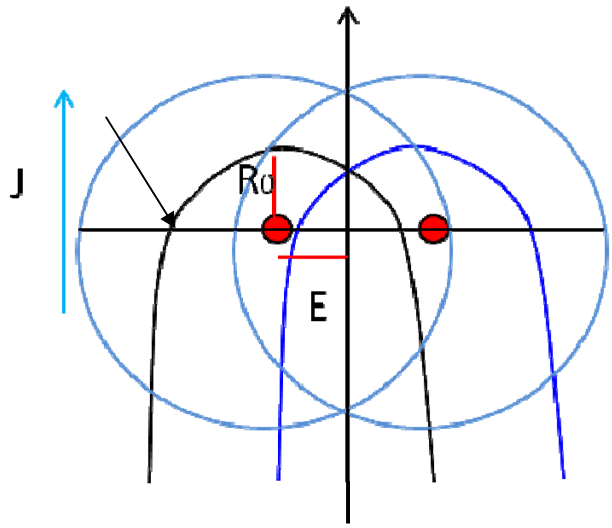

Fig. 6 – Configuration of two aligned wells and origin of a stagnation point

In Fig. 6 j' and j'' are two vectors which sum is –J (1.2a):

$$J' + J'' = -J \tag{1.2a}$$

If j' = j'' = j, when -2j = J there is a stagnation point.

The velocity J is equal everywhere, because the groundwater is homogeneous; it is simple to note that condition for jA = jB opposite to J, which generates the stagnation point, are only along the Y axis, when (as shown in Fig. 6) occur the conditions for:

$$J_A \cos b = 0{,}5J \tag{1.2b}$$

To identify where those conditions occur, considering the specific case of two wells, the problem solution is simple. At first, a fictitious single well influence radius R0, corresponding to the stagnation point on the vertex of the watershed must be determined.

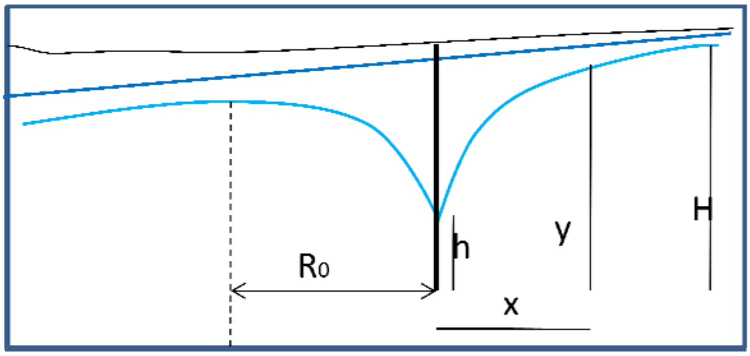

Fig. 7 – Representation of variable in (1.3): h is the hydraulic head in correspondence with one extraction well, y the hydraulic head at assigned distance x from the same well, and R_0 the fictitious radius of the well

The stagnation point of the unique well is located in correspondence with the maximum of the piezometric profile function, where the piezometric head y is:

$$y = h + \frac{Q}{2\pi T} ln\left(\frac{x}{r}\right) - xJ$$

(1.3)

where h [m] is the piezometric level in the well which extracts the constant flow Q, r [m] is the influence radius of the well and J [-] is the piezometric gradient of groundwater.
The function which allows to calculate y is then:

$$y = h + \frac{Q}{2\pi T} ln\left(\frac{R_0}{r}\right) - R_0 J$$

(1.4)

where x [m] is the distance from the well where the derivative is equal to 0:

$$x = \frac{Q}{2\pi TJ} \qquad \textbf{(1.5)}$$

The x value is equal to a maximum of the piezometric level function, and is named fictitious influence radius R_0 (Fig. 7); along the piezometric profile which links this point to the upper well, the water flows towards the well, elsewhere, passed the well, water flows to the same delivery of the groundwater. This watershed point represents a zero – flow point, in the case of only one pumping well, or two equal wells, pumping the same Q, when depressions do not superimpose in R_0.
Each point of the motion field of the groundwater, where the distance from one of the two wells is equal to (1.5), is a stagnation point.

In Fig. 8 the influence radius and the vectors of piezometric gradients due to two aligned wells are shown.

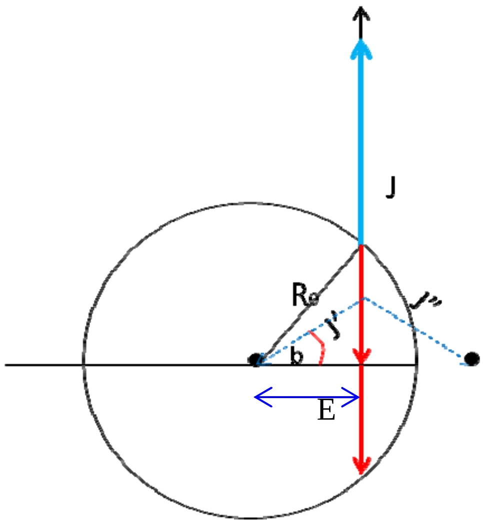

Fig. 8 – Gradient J of the groundwater and gradients j' and j'' created by the withdrawal Q from both the wells, compared to the fictitious influence radius

When there are two wells which pump the same rate in a homogeneous aquifer (Fig. 4), a j value identical to the J of the groundwater is obtained in the points within a distance of R_0 from both the wells, positioned in order to obtain the sum of vector representing the piezometric gradient equal to J:

$$(jA + jB)\cos b = J$$

(1.6)

The value of b angle is unknown, depending from J, R_0 and E.

3 ANALYTICAL APPROACH TO LOCATE STAGNATION POINTS: AN EXAMPLE OF APPLICATION OF THE COMPLEX POTENTIAL THEORY TO A REAL CASE

By the solutions presented by Shan (1999), Christ and Goltz (2004), a new model for lot of wells has been developed (Colombo et al., 2012, *in press*). This model considers not only a number of wells N ≥ 2, but also the possibility to locate the wells everywhere in the complex plain *(x, y)*. To use this models, any simplifications are necessary: the aquifer must be homogeneous, isotropic, confined, with an uniform B thickness and constant Darcy velocity J. The flux is stationary. The complex potential w (Javandel and Tsang, 1984), by the linearity of Laplace equation, can be expressed as the superimposition of effects of a system formed by N injection or pumping wells and the groundwater. It is:

$$w = \phi + i\psi = -Jze^{-i\alpha} + \sum_{j=1}^{N} \frac{Q_j}{2\pi B} ln(z - z_j) + C$$

(1.7)

Where w is the complex potential of the entire system, J [m/s] is the constant Darcy velocity, α is the angle between the water flow and the x axis, B [m] is the thickness of the aquifer, Q_j [m^3/s] is the extraction rate of the well if ≥ 0, N is the number of considered wells, z (z = x + iy) is the position in the complex plain where the potential w is evaluated, z_j ($z_j = a_j + ib_j$) is the position in the complex plain (x,y) of the well j, where a [m] and b [m] are the coordinates of the well in the complex plain, $i = \sqrt{-1}$, C is a constant integration value which depends by the boundary conditions.

The complex potential w can be divided in a real part and in an imaginary one; the real part *Φ* represents the lines with the same potential hydraulic head and is done by:

$$\phi = -J(x\cos\alpha + y\sin\alpha) + \sum_{j=1}^{N}\frac{Q}{4\pi B}ln\left[(x-a_j)^2 + (y-b_j)^2\right]$$

(1.8)

The imaginary part *Ψ* represents the function of the flow:

$$\Psi = J(x\sin\alpha - y\cos\alpha) + \sum_{j=1}^{N}\frac{Q_j}{2\pi B}tan^{-1}\left(\frac{y-b_j}{x-a_j}\right)$$

(1.9)

To obtain the formulation of the capture curve function, it is necessary at first to evaluate the function of the flux in the stagnation point, corresponding to velocity equal to 0. The stagnation point can be calculated (Christ and Goltz, 2002) using the derivative in the complex field the potential w respect to z and set the obtained equation equal to 0.

$$\frac{dw}{dx} = -Je^{-i\alpha} + \sum_{j=1}^{N}\frac{Q_j}{2\pi B(z-z_j)}$$

(1.10)

For example, a 5 wells barrier (as in Fig. 5), is described by an equation of 5th degree.

$$i\sin\alpha z^5 + \beta z^4 - \gamma i z^3 - \delta z^2 + \varepsilon i z + \in = 0$$

(1.11)

in which the complex coefficients are:

$\beta = 2b\sin\alpha + 5R_0, \gamma = b^2\sin\alpha + 5a^2\sin\alpha + 8R_0 b, \delta = 15a^2R_0 + 3b^2R_0 + 8a^2b\sin\alpha, \varepsilon = 16a^2bR_0 + 4a^4\sin\alpha + 4a^2b^2\sin\alpha,$
$\in = 4a^4R_0 + 4a^2b^2R_0$

These values are dependent by the barrier geometry (coordinates a and b of each well), the angle of the groundwater flow α and the influence radius R_0 [m].

The identification and study of stagnation points and hydrogeological structures which contribute to the genesis of the same points, appears to be a study of interests.
The identification and calculation of secondary stagnation point has been simple and easy. This secondary point forms on the axis between two identical pumping wells, extracting the same flow by a homogeneous aquifer. In this case, in fact, the stagnation point appears where the sum of gradients of the flow towards wells is equal and opposite of the groundwater flow gradient. This phenomenon occurs on the axis within a distance equal to the fictitious influence radius of each well, after the line which joins the wells. Due the fact that capture zone goes past the secondary stagnation point and the principal ones, the possibility to calculate easily their position represents an ulterior way to control barriers using piezometric data.

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